PRACTICAL NONLINEAR INELASTIC ANALYSIS METHOD OF 3D COMPOSITE STEEL-CONCRETE FRAMEWORKS

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Abstract: A second-order flexibility based model has been developed for advanced analysis of 3D composite steel-concrete frameworks with partial composite action. Both distributed plasticity and element geometrical effects are modeled by combining the Maxwell-Mohr rule and the second-order force based functions for the derivation of the force-displacement relationship at the element level. The proposed model allows efficient modeling of the combined effects of nonlinear geometrical effects, spread-of-plasticity, partial shear connection of composite beams, finite-size joints and joint flexibility by using only one 2-noded beam-column element per physical member. The proposed nonlinear inelastic analysis formulation has been implemented in a general nonlinear static purpose computer program, NEFCAD. An advanced FEM model has been developed in Abaqus software considering a combination of three-dimensional solid elements (for concrete volumes) and shell elements for steel elements allowing also the partial composite action between the steel beam and concrete slab. Several computational examples are given to validate the accuracy and reliability of the proposed method.

Keywords: Nonlinear inelastic analysis; Flexibility-based element; Distributed plasticity; Partial composite action; Advanced analysis

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1 INTRODUCTION

In recent years, have witnessed significant advances in nonlinear inelastic analysis methods for composite steel-concrete beams and framed structures and integrate them into the new and more rational advanced analysis and design procedures [1, 2, 3]. However, in order to allow the partial composite action in the case of composite beams the majority of the available methods include additional degrees of freedom at the element ends, thus the computational effort is greatly enhanced in the case of large scale frame structures. Furthermore, when modelling the semi-rigid composite frameworks some difficulties may arise enforcing the compatibility conditions at the semi-rigid composite connections [3, 4]. In spite of the availability of such nonlinear inelastic algorithms and powerful computer programs, the nonlinear inelastic analysis of real large-scale frame structures with partially connected members still possess high demands on the most powerful computers available and still represents unpractical tasks to most designers.

The present work attempts to develop accurate yet computational efficient tools for the nonlinear inelastic analysis of partially connected composite steel-concrete frameworks fulfilling the practical and advanced analysis requirements.
2 MATHEMATICAL FORMULATION

The following general assumptions are adopted in the formulation of the proposed analytical model for inelastic analysis of composite beams: (1) Plane sections remain plane for entire cross-section after flexural deformation; throughout the depth of the cross-section, the strain distribution is linear, but a discontinuity exists at the concrete slab-steel beam interface due to slip, frictional effects and uplift are neglected, the interface slip is small; (2) The vertical displacement and the curvature of the different subcomponents (concrete slab and structural steel) are assumed to be the same; (3) Discretely located interlayer connectors with uniform spacing are regarded as continuous and ductile with an nonlinear elastic-plastic behavior. In the formulation of the inelastic behavior of composite columns full composite action between concrete matrix and steel profile is assumed. Small strains but large displacements and rotations can be considered. Transverse shear deformations, associated to the transverse shear forces are neglected in the plastic constitutive relationships. The model suggested by the CEB-FIB Model Code 90, is adopted in the present paper to model the concrete under compression and tension. Multi-linear elastic-plastic stress-strain relationships, both in tension and in compression, are assumed for the structural steel and the conventional reinforcing bars. Gradual yielding of cross-sections and elasto-plastic tangent flexural and axial rigidities, for composite steel-concrete columns with arbitrary shape, when full shear connection is assumed, is described through basic equilibrium, compatibility and material nonlinear constitutive equations following the procedure described in [3] where the discussions concerning the numerical integration of stresses and stiffnesses over cross sections and inclusion of residual stresses for structural steel are also addressed. Therefore, this paper is focused on partial composite action effect over inelastic response of beam cross-sections. As will be briefly described in the following sections, the incremental force-displacement relationships at the element level are derived by applying the Maxwell-Mohr rule for computation of generalized displacements in the second-order geometrically nonlinear analysis and using an updated Lagrangian formulation the nonlinear global geometrical effects are considered updating the element forces and geometry configurations at each load increment [4, 5].

2.1 Elasto-plastic analysis of beam cross-sections

The elastic and inelastic behavior of steel-concrete composite beams is quite complex because the shear connectors generally permit the development of only partial composite action between the individual components of the member, and their analysis requires the consideration of the interlayer slip between the subcomponents. Usually, for a given composite beam, the full shear connection is defined as the least number of shear connectors \( n_f \) such that the bending resistance of the beam would not be affected if more shear connectors are provided, whereas partial shear connection occurs when the number of connectors \( (n) \) used in a beam is lower than \( (n_f) \). In order to analyze and design the composite beams, simplified methods are very useful and such methods are proposed in international literature and in some codes. For instance in the Eurocode 4 the concept of the degree of shear connection \( \eta = n/n_f \) is used and the ultimate bending strength capacity of cross-section is evaluated by simple equilibrium of stresses with a prescribed compressive axial force in the concrete slab \( N_c = nP_{sc} \), where \( n < n_f \) represents the number of shear connectors and \( P_{sc} \) is the connector strength.
The composite beam cross-section considered here consists of a concrete solid slab connected to a steel beam as presented in Figure 1. The inelastic response of the composite beam under the assumption of the full composite action (Fig. 1a) can be described through basic equilibrium, compatibility and material nonlinear constitutive equations as described in [4]. Hence, the internal axial force in concrete slab ($N_{cf}$) in which the contribution of the conventional steel reinforcements is included, can be evaluated [4]. Let us consider now the cross-section, in partial composite action, subjected to the action of the external bending moment ($M$) and axial force ($N$), as shown in Figure 1b. Under the above assumptions, the resultant strain distribution, corresponding to the curvature $\phi$ and the axial strains $u_c$ and $u_s$ evaluated at the centroid of concrete slab and structural steel respectively, can be expressed in a linear form as:

$$
\varepsilon_c = u_c + \phi \cdot (y - r);
\varepsilon_s = u_s + \phi \cdot y
$$

where $\varepsilon_c$ and $\varepsilon_s$ represents the strains in concrete slab and steel beam respectively and $r$ represents the distance from the central axis of the concrete slab to that of the steel beam. The equilibrium is satisfied when the external forces are equal to the internal ones:

$$
\begin{align*}
\int_A \sigma_c(u_c, \phi) dA_c + \int_A \sigma_s(u_s, \phi) dA_s + \sum_{i=1}^{N_s} \sigma_i(u_c, \phi) A_{r_{ei}} - N &= 0 \\
\int_A \sigma_s(u_s, \phi) y dA_s + \int_A \sigma_c(u_c, \phi) y dA_c + \sum_{i=1}^{N_s} \sigma_i(u_c, \phi) y_A_{r_{ei}} - M &= 0
\end{align*}
$$

in which $u_c$, $u_s$ and $\phi$ represent the unknowns. In order to solve the above nonlinear system, the internal axial force in the concrete slab ($N_{int}$), under partial composite action, is assumed to be a fraction of the axial force in the concrete slab under full composite action ($N_{cf}$) and the amount is defined by a function of the degree of composite action $\gamma_{eff}$ as in Figure 1b:

$$
N_{int}^c = f(\gamma_{eff}) \cdot N_{cf} = N_{c}^c
$$
where $N_{i,c}$ represents the internal axial force in the concrete slab of the cross-section subjected to the same external bending moment $M$ and axial force $N$ but under the assumption of full composite action between the steel beam and concrete slab, $f(\gamma_{eff})$ represents a function of the effective degree of composite action [4]. Thus, the nonlinear system (4) becomes:

$$\int_{A} \sigma_i' (u_i, \phi)dA_i + \sum_{i=1}^{N_i} \sigma_i (u_i', \phi)A_{ni} - N_{i,c} = 0$$

$$\int_{A} \sigma_i (u_i, \phi)dA_i - (N - N_{i,c}) = 0$$

$$\int_{A} \sigma_i (u_i, \phi)ydA_i + \int_{A} \sigma_i' (u_i, \phi)ydA_i + \sum_{i=1}^{N_i} \sigma_i (u_i', \phi)y'A_{ni} - M = 0$$

The above system can be solved numerically using the Newton iterative method. The incremental relationships between incremental efforts and incremental deformations can be expressed as:

$$\begin{bmatrix} k_{11} & 0 & k_{13} \\ 0 & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \Delta u_c \\ \Delta u_x \\ \Delta \phi \end{bmatrix} = \begin{bmatrix} \Delta N_{c} \\ \Delta N_x \\ \Delta M \end{bmatrix}$$

(5)

where the coefficients of the tangent stiffness matrix $k_{ij}$ can be evaluated as in [4]. We define the tangent flexural rigidity of cross-section as a ratio between incremental bending moment and incremental curvature while keeping constant the axial force ($\Delta N = 0$), the tangent flexural rigidity of the cross section with partial composite action can be developed as [4]:

$$\frac{(EI)}{a} = \frac{a}{1 - bf(\gamma_{eff})}\frac{\Delta N_{cf}}{\Delta M}$$

$$a = k_{33} - \frac{k_{31}k_{13}}{k_{11}} - \frac{k_{32}k_{23}}{k_{22}}; \quad b = \frac{k_{31}}{k_{11}} - \frac{k_{32}}{k_{22}}$$

where $\Delta N_{cf}$ represents the incremental axial force in the concrete slab under the assumption of full composite action computed as a difference between the axial force in the concrete slab associated at given value of the bending moment $M$ and the axial force associated at an incremented bending moment $M + \Delta M$ [4]. The value of effective degree of composite action defined in [4] is assumed to be constant over the length of the member and is updated at each loading step according with the existing state of stress [4].

Need to be mentioned that for shear forces ($P$) less than 50% of $P_{sc}$ a constant value for the shear connection stiffness ($k_{50\%}$) is considered (Figure 2), assuming a secant connector stiffness corresponding to 50% of $P_{sc}$ while for shear forces greater than 50% of $P_{sc}$, a secant value for the shear connection stiffness ($k_{sec}$) is considered (Figure 2) [4].

As already mentioned above the approach discussed in this paper assume for the entire member length a constant value for the function used to introduce the partial composite action $f(\gamma_{eff}) = constant$. Hence the effects of non-uniform distribution of the shear connectors cannot
be taken into account and also the accuracy in detecting stress distribution along the member length can be only determined in an approximately manner. A more efficient approach able to overcome the above mentioned drawbacks is currently under development. Such an approach implies an explicit solution of the second-order differential equilibrium equation of the composite beam with partial composite action in which the axial force in the concrete slab represents the main unknown.

![Nonlinear constitutive law for the shear connection.](image)

This axial force, the solution of the differential equilibrium equation, can be expressed in function of the axial force under the assumption of the full composite action multiplied with a function of the degree of composite action which includes also the exact distribution of the bending moment along the member length. Moreover, by simply dividing the beam according with the variable distribution of shear connectors and solving the second-order differential equilibrium equations for each segment considered as beams with uniform distribution of shear connectors, and then imposing the boundary and continuity conditions could represents a direct and simple way to extend the proposed approach to the cases of non-uniform distribution of shear connection along the beam length. In this way both uniform and non-uniform distribution of the shear connectors can be efficiently considered in the proposed formulation but these issues requires further investigations and calibrations and will be treated in a future work.

### 2.2 Second-order flexibility-based element

Flexibility-based method is used to formulate the distributed plasticity model of a 3D frame element (12 DOF) (Figure 3). The spread of inelastic zones within an element is captured considering the variable section flexural $EI_y$ and $EI_z$ and axial $EA$ rigidity along the member length, depending on the bending moments and axial force level, cross-sectional shape and nonlinear constitutive relationships as already described.

The basic incremental force-displacement relationships are determined considering the element represented in natural coordinate system with rigid body modes removed [4, 5]. The element incremental flexibility matrix $f$, which relates the end displacements to the actions $\Delta s$, can be derived by applying Maxwell-Mohr theorem for computation of generalized displacements [4]:
Assuming elastic behavior within a load increment, and no coupling of axial and flexural responses at the section level, the generalized displacement in point $i$ of the member produced by the force $\Delta P$ ($\Delta M_{iy(z)}, \Delta M_{ijy(z)}, \Delta T_{iy(z)}, \Delta T_{ijy(z)}$) could be expressed as in equation (8) where with superior indices (I) and (II) represent the first order and second order efforts respectively. The second term in equation (8) introduces the additional effect of shear deformations. In this respect, for composite beams with partial composite action, equivalent transverse shear stiffness has been derived by using the energy relations [4]. For column cross-sections equivalent transverse shear stiffness is assumed to be computed taken into account only the contribution of the steel component. The second order bending moments and shear forces can be evaluated in function of the nodal bending moments and uniform distributed loads as described in [5] and then the relationship between nodal displacements ($\Delta u_r$) and the nodal efforts ($\Delta s_r$) could be further expressed by defining the element flexibility matrix ($f_r$) and nodal displacements given by the uniform distributed loads ($\delta_r$) as:

$$\Delta u_r = f_r \Delta s_r + \delta_r$$  

(9)

The detailed expressions for $f_r$, $\delta_r$ can be found in [5]. To produce the deformational-stiffness relation, equation (9) is inverted, obtaining the following deformational-stiffness equation:

$$\Delta s_r = k_r \Delta u_r - \Delta q_r;$$

$$\Delta q_r = k_r \delta_r$$  

(10)

where the vector $\Delta q_r$ is the incremental equivalent load vector, whereas $k_r$ represents the instantaneous element stiffness matrix of the beam-column element without rigid body modes,
determined by matrix inversion of the flexural matrix $f$. This element stiffness matrix may be further extended by inclusion the effects of finite joint sizes and semi-rigid connections following the procedure developed in [6]. The resulting stiffness matrix is a 4x4 matrix, and does not include torsional and axial degrees of freedom. Torsional and axial stiffness components are then added to result in the required 6x6 stiffness matrix. To include rigid body modes, the stiffness matrix is pre- and post-multiplied by a transformation matrix to result in the required 12 x12 matrix [4, 5].

2.3 Advanced finite element modelling of composite steel-concrete structural elements

In this study, the advanced numerical simulation is conducted with ABAQUS v.6.11 [7] software, which is able to take into account the main factors that govern the structural behaviour of steel-concrete composite beams with full and partial composite action. In the following subsections, modeling details related to material constitutive models, type and dimensionality of selected finite elements, shear studs modeling, analysis procedures are presented.

2.3.1 Material modelling

The elastic branch of concrete under compression is defined by Young’s modulus and Poisson’s coefficient and its limit stress is chosen as $0.4 f_{cm}$ where $f_{cm}$ is taken as the actual cylinder strength test value. The inelastic behaviour of concrete is defined with CDP-Concrete Damaged Plasticity model which assumes that the two main failure mechanisms are tensile cracking and compression crushing of concrete material. The dilatation angle and the viscosity parameter are taken as 38° and 0.001 respectively, while default values are used for all other parameters that define the plasticity model. The structural steel and reinforcing steel is modelled as an elastic-plastic-linear hardening material, using the classical metal plasticity model through *Plastic option, which assumes Mises yield surface and allow for isotropic hardening behaviour.

2.3.2 Element types and mesh definition

In order to properly select the analysis model and the associated finite element types of composite beams, three numerical models hereinafter referred to as A, B, C, are proposed and verified. The main differences between these three models consists in the finite element types selected to simulate each component of the composite system, as shown in table 1 below. It can be seen that eight-node-incompatible mode $C3D8I$ elements are selected to represent the concrete part of each model, these elements being able to successfully manage hourglassing and shear locking phenomenon. Because of the added internal degrees of freedom these elements are more computationally expensive than the regular first-order $C3D8$ and $C3D8R$ (reduced integration) elements but significantly more economical than second-order $C3D20$ and $C3D20R$ elements [7]. In model B, the steel joist is represented by means of four-node, general purpose reduced integration (one integration point) shell element $S4R$, which takes into account the effect of shear deformations, thus providing accurate solutions to both thin and thick shell problems while in model C, two-node three-dimensional shear flexible beam elements, $B31$, with linear interpolation are selected to model the structural steel. The reinforcing bars are defined as two-node three-dimensional linear truss elements, $T3D2$, which assume linear interpolation and constant stress over the element length. The shear connectors are explicitly modelled in case A using $C3D8I$ brick elements, whereas special purpose connector elements $CONN3D2$ are adopted to define the steel – concrete connection in numerical models B and C. The connector elements have zero length in model B and half of the steel profile height in model C. In this study, Cartesian + Align connector type is
selected, hence the connector second’s node can independently translate in three local cartesian directions while the rotations are prohibited.

Table 1: Finite element families used in the proposed numerical models for composite beams.

<table>
<thead>
<tr>
<th></th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete slab</td>
<td>solid C3D8I</td>
<td>solid C3D8I</td>
<td>solid C3D8I</td>
</tr>
<tr>
<td>Steel profile</td>
<td>solid C3D8I</td>
<td>shell S4R</td>
<td>beam B31</td>
</tr>
<tr>
<td>Reinforcements</td>
<td>truss T3D2</td>
<td>truss T3D2</td>
<td>truss T3D2</td>
</tr>
<tr>
<td>Shear connectors</td>
<td>solid C3D8I</td>
<td>connector CONN3D2</td>
<td>connector CONN3D2</td>
</tr>
</tbody>
</table>

Figure 6: Proposed numerical models

For the two in-plane translational directions the relative behaviour is defined by a force-displacement constitutive law, while a rigid connection is assumed in a direction normal to the steel-concrete interface (the uplift is neglected). The constitutive law, that describes the in-plane translational behaviour, is usually obtained from experimental push-out tests on shear studs, or, alternatively, when experimental data are not available, it can be described by Ollgaard’s model as already shown in Figure 2. The mesh size significantly influences the computational effort and the accuracy of the results. Concerning this, sensitivity studies have indicated that mesh dimensions aspect ratio close to 1.0 and an average size of 50 mm for
concrete slab and steel profile provides accurate results. The shear studs in model A were meshed in solid elements with maximum size of 5 mm. Inspecting the described numerical models, it can be concluded that the most refined but computationally expensive model is A.

2.3.3 Validation of the proposed numerical models

The proposed numerical models are validated by comparison against Chapman and Balakrishnan [8] experimental tests on simply supported composite beam E1. Details about geometry and material properties can be found in [4, 8]. Figure 6 shows the three numerical models generated based on the abovementioned details. The analyses are conducted with the nonlinear static arc-length based Riks procedure. This method can provide solutions even in cases of complex, unstable response or, where, the load-displacement response exhibit negative stiffness [7].

![Figure 6: Composite beam E1: a. Load-deflection curves; b. Slip distribution](image)

Figure 7: Composite beam E1: a. Load-deflection curves; b. Slip distribution

Figure 7a presents the comparative load versus mid-span deflection curves obtained by the proposed finite element numerical models in conjunction with experimental results retrieved from [8]. As can be seen, model A predicts with high accuracy the initial stiffness of the composite beam, but overestimates the ultimate load capacity of the system, most likely due to numerical problems exhibited by solid finite elements. Running on a computer with two 2.80 GHz clock speed CPUs (having eight cores) and 48 GB RAM, the present analysis is performed in approximately 7 hours. On the other hand, the curve predicted with model B is in very close agreement with the experimental one and it’s generated in 10 minutes on the
same computer. Finally, it can be observed, that model C slightly underestimates the initial stiffness and the ultimate load capacity while the analysis time is slightly lower than that needed by model B. The slip distribution along beam E1 corresponding to a load level of 450 kN is shown in figure 7b. In the numerical models, the slip was evaluated, at the steel-concrete interface, as the horizontal relative distance between the steel profile and the concrete slab, of each positioning section of the studs. Consequently, the slip distribution in model C cannot be recorded because the displacements of the steel profile are calculated only on the reference axis of the beam (see Figure 6). The slip distribution predicted with model B is in good agreement with the experimental one, while model A exhibits increased rigidity.

Comparing the computational efficiency and accuracy of the proposed numerical models, the second one is selected and it is used further on in this study.

3 COMPUTATIONAL EXAMPLES

The accuracy of the numerical procedure developed here and implemented in the computer program (NEFCAD) has been evaluated using several benchmark problems. In the present approach, one element has been used to model each column and beam in all computational examples and the advanced numerical simulation is conducted by using the specialized software for nonlinear analysis of structures, ABAQUS.

3.1 Elasto-plastic cross-sectional analysis

Due to the fact that in the proposed approach the key elements of the elasto-plastic formulation are established at the cross-sectional level, the first set of numerical experiments is conducted at the cross-section level. In order to prove the reliability, efficiency and numerical stability of the proposed approach several moment-curvature analyses have been performed considering the material and shear connection properties of beam cross-section A6 used in the experimental tests by Chapman and Balakrishnan [8]. The geometry and material properties of the tested cross-section can be found in [4, 8]. The bending moment – curvature comparative curves are plotted in Figure 8a. As it can be seen the behaviour of the composite cross-section predicted by the proposed analysis procedure is in close agreement with that of experimental test. The results are also consistent with those obtained by Ban & Bradford [9] with an analytical method in which the partial shear connection is defined at cross-sectional level assuming the slip strain at the steel-concrete interface to be a function of the degree of shear connection.

Figure 8b shows the comparative bending moment-curvature diagrams of A6 cross-section, considering different values for the degree of composite action by means of adopting different values for the function \( f(\gamma) \) ranging from \( f(\gamma)=1 \) (i.e. full composite action) and \( f(\gamma)=0 \) (i.e. no composite action). It can be observed that, as expected, on decreasing the level of shear connection (i.e. decreasing the values of \( f(\gamma) \)) the strength and initial stiffness of the cross-section is reduced.
3.2 Six-story composite frame

The geometry loading conditions of Vogel’s six-story two-bay frame are reported in [4]. The yield strength of all steel members is 235 MPa while Young and shear Modulus are E=20500 MPa and G=7885 MPa. The compressive yield strength of concrete for columns is $f_c=26$MPa and for concrete slab $f_c=16$MPa. In order to evaluate the effects of the finite size of the joints and to make possible comparisons with more advanced nonlinear FEM solutions (Abaqus), in the proposed approach (Nefcad) the frame has been modelled considering member end offsets and assuming effective rigid joint size of one-half the true size of the joint. In this study a value of $\gamma_{sc}=64.34$ kN was considered for the shear connector capacity. The parameters that describe the shape of the studs constitutive law have been selected as proposed by Ollgaard: $\beta=1$ and $\alpha=0.558$, obtaining in this way a value of 94375 N/mm for the connector stiffness corresponding to 50% of $\gamma_{sc}$, namely $K_{50\%}$ (see figure 2). The values of shear connection stiffness, $k_{50\%}$, are evaluated based on the number of shear connectors corresponding to the desired degree of shear connection [4].

Figure 7a presents the comparative load-deflection curves obtained by the proposed approach when full composite action is assumed and those obtained with the advanced FEM model considering different levels of shear connection, ranging from 100% to 300%. It can be observed that, as expected, increasing the level of shear connection the system became more rigid with increased strength and stiffness and the inelastic behavior becomes similar with the response predicted by the proposed approach (full composite action). The effectiveness of the proposed procedure is further assessed by varying the degree of shear connection and comparing the predicted curves with those obtained with more complex finite element analysis, as shown in figure 7b.

4 CONCLUSIONS

A reliable and robust nonlinear inelastic analysis method for composite steel-concrete frames with partial composite action has been developed. The proposed formulation has been found to be effective in predicting the global behavior of composite beams and frames with partial shear connection, both in elastic and post-elastic field. The numerical results agree fairly well with the experimental results and those obtained by advanced nonlinear FEM approaches but with much less computational effort.
The proposed formulation takes advantage of using only one 2-noded beam-column element with 6 DOF for modeling the element geometrical effects, distributed plasticity and partial composite action in the case of composite beams, featuring in this way, the ability to be used for practical applications by combining modeling benefits, computational efficiency and reasonable accuracy. Future work is envisaged, considering the extension of the proposed method for nonlinear inelastic analysis of 3D composite steel-concrete frames by including the effects of the inelastic behavior of composite beam-to-column joints with panel zone deformations.

Figure 9: Load-deflection curve for six-story composite frame

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