STUDY ON SLAB TRANSVERSE MOMENT DISTRIBUTION IN TWIN GIRDER CROSS-BEAM COMPOSITE BRIDGE

Da Xiang\textsuperscript{1*}, Yuqing Liu\textsuperscript{1}, Xiaqing Xu\textsuperscript{2}

\textsuperscript{1}Department of Bridge Engineering, Tongji University, Shanghai, China
Emails: 1351291@tongji.edu.cn, yql@tongji.edu.cn

\textsuperscript{2}College of Civil Engineering, Chongqing University, Chongqing, China
Email: xq.xu@foxmail.com

\textbf{Abstract:} In twin girder cross-beam composite bridges, the structural characteristics of steel girders may have effect on the transverse moment distribution of concrete slabs. In this paper, finite element analysis of an actual composite bridge was conducted to study the transverse moment distribution of concrete slabs subjected to a linear uniform load. The impacts of the spacing of headed studs at the steel-concrete interface were also investigated. Then a frame model constrained by springs was introduced to explore the mechanism of the transverse moment distribution of concrete slabs. The results show that the transverse flexural stiffness of steel girders, mainly determined by the layout of crossbeams and the vertical stiffeners, is the major influencing factor on the moment distribution. The spacing of headed studs also slightly affects the moment distribution since the connectors could change the rotational constraints on slabs provided by steel girders. The comparison between FEA results and the frame model method proves that the proposed frame model could obtain accurate results for predicting the transverse moment distribution.

\textbf{Keywords:} Steel-concrete composite bridge; Transverse moment distribution; Finite element analysis; Frame model

\textbf{DOI:} 10.18057/ICASS2018.P.041

\section{INTRODUCTION}

The deck of twin girder cross-beam composite bridge is a concrete slab, which is supported by a steel frame comprising two main girders connected to the deck slab and interlinked by secondary beams called cross-beams that are at no point in contact with the slab [1]. Owing to the superior mechanical property and construction convenience, the twin girder cross-beam composite bridges have been widely used in recent years [2,3]. At present, most researches focus on the mechanical behaviors of steel girders and the steel-concrete connectors [4-7]. While researches on the transverse moment distribution of concrete slabs supported by steel girders under live load are rarely reported. The concrete slab is connected to the upper flange of steel girders through shear connectors like headed studs. As shown in Figure1, when the slab is loaded at transverse midspan, the negative bending moment $M_x$ would be generated in the slab near the support and the positive bending moment $M_e$ near the midspan. When the slab is loaded at cantilever, a part of negative moment $M^c_x$ would be transferred to midspan slab, namely $M^c_e$. The distribution of above moment is determined by the rotation constraints on concrete slabs, which are closely related to the transverse bending stiffness of
steel girders. Meanwhile the transverse bending stiffness of steel girders is mainly depended on the layout of crossbeams and the vertical stiffeners.

Generally, references for the design of bridge slabs (continuous slabs) includes Chinese code (JTG 3362-2018) [8], or Japanese Code (Specifications for Highway Bridges) [9] and AASHTO LRFD specification [10]. In these codes, the value of $M_c$ and $M_s$ are suggested to be calculated as $-0.7M_0$ ($M_0$ represents the maximum positive moment at midspan of a simply supported slab with the same effective span $l_0$ as the slab being analysed) and $0.7M_0$ in Chinese code, $-0.8M_0$ and $0.8M_0$ in Japanese Code and AASHTO LRFD specification, respectively. However, these recommendations fail to take the structural characteristics of steel girders into consideration and the boundary conditions of slabs in twin girder cross-beam composite bridges may be different from continuous slabs. In addition, there is no available reference on the methods to calculate the value of $M_c^\epsilon$. However, researches on the transverse moment distribution of slabs loaded at midspan and cantilever is of great significance to optimize the design of these slabs.

![Figure 1: Transverse moment distribution of slabs under live load](image)

In this paper, finite element analysis was conducted on the basis of an actual twin girder cross-beam composite bridge to study the transverse moment distribution of the slab loaded at midspan and cantilever. The impacts of spacing of headed studs at the steel-concrete interface were investigated. Further, in order to explore the mechanism on how steel girders affect the moment distribution of the slab, a simplified frame model with springs was introduced based on the basic principles of frame analysis.

## 2 FINITE ELEMENT ANALYSIS

### 2.1 Finite element model

Based on the dimensions of a twin girder cross-beam composite bridge in Shanghai (Figure 2), a simply supported steel-concrete composite bridge model with a span of 25m is established using ABAQUS. The dynamic explicit method was used. The longitudinal spacing of vertical stiffeners and crossbeams of steel girders are 2m and 4m, respectively. As shown in Figure 3, only one quarter model of the full bridge is simulated considering the symmetry in the longitudinal (Z) direction and transverse (X) direction, so the symmetric boundary conditions are used on relevant surfaces.

The uniaxial stress-strain relationship of concrete is obtained from the Chinese code: GB50010-2010 [11] with Young’s modulus of 34.5GPa. The compressive cylinder strength ultimate and tensile cylinder strength of concrete are 35MPa and 2.6MPa respectively. For simplicity, a bi-linear stress-strain curve with no strain hardening is used to model the stress-strain relationship of steel and reinforcement with initial Young’s modulus of 200GPa.
and yield strength of 400MPa. The mechanical behavior of steel and reinforcement for both tension and compression is assumed to be similar. The steel members and the concrete slabs are all simulated by the 8-node linear hexahedron reduced integration element C3D8R, whose edge length is 0.05m. The reinforcement is simulated by truss element T3D2 and is embedded into the concrete elements. It is assumed that there exists no slip between steel girders and concrete slabs and the “tie” constraint is adopted to make the nodes at the steel-concrete interface coupled [12]. To simulate the live load and make concrete slabs uncracked, linear uniform loads with the value of 20kN/m are longitudinally loaded at midspan and cantilever. The distance between the center lines of load at midspan and of web is 1750mm while the distance between the center lines of load at cantilever and of web is 1850mm.

![Figure 2: Cross section of model(unit:mm)](image2.png)

![Figure 3: Constraints and loads of model](image3.png)

### 2.2 Transverse moment distribution

To obtain the transverse moment distribution of slabs along X direction, the cross section containing cross-beam (section I), the cross section only containing vertical stiffener (section II) and the unstiffened cross section (section III) (Figure 4(a), (b), (c)) are selected as three key sections. Transverse (x-direction) stresses at the top of the slabs of three key sections are obtained and converted into equivalent moment by the equation: $M = \sigma I / y$ (the width of section is assumed as 1m), which are shown in Figure 4(d) and Figure 4(e).

As shown in Figure 4(d), when slabs are loaded at midspan, three key sections have the same maximum positive moment $M_{z}$, but the maximum negative bending moment $M_{s}$ is increasing gradually under the order of section III, II, I. Considering the transverse flexural stiffness of steel girders increases under the order of section III, II, I, it is indicated that the moment distribution is greatly influenced by transverse flexural stiffness of steel girders and the maximum negative bending moment $M_{s}$ has an increasing tendency with the enlargement of the transverse flexural stiffness of steel girders.

As shown in Figure 4(e), when slabs are loaded at cantilever, three key sections have the same maximum cantilever moment $M_{z}^c$ and negative midspan moment $M_{c}$, but the $M_{s2}$ at $x=1.9$m is decreasing gradually under the order of section III, II, I, which indicates a part of negative moment generated by the live load at cantilever is transferred into steel girders from concrete slabs and transferred moment increases with the enlargement of transverse flexural stiffness of steel girders.
To investigate the distribution of the transverse moment along the Z-direction, $M_y$, $M_c$, $M_i$, $M_{i2}$ and $M_c$ are obtained along Z-direction as shown in Figure 5. When slabs are loaded at midspan, $M_y$ reaches a peak value near cross-beams and vertical stiffeners. But such increase only occurs in a limited range in the Z-direction whose center is at the location of cross-beam or vertical stiffener. However, there is no obvious change of $M_c$ along the Z-direction. And when slabs are loaded at cantilever, only $M_{i2}$ decreases in a limited range around cross-beam and vertical stiffener in the Z-direction.
2.3 Moment proportion

In order to investigate the moment proportion, $M_c/M_0$, $M_s/M_0$ and $M_c^c/M_0^c$ ($M_0^c$ represents the maximum cantilever negative moment of a cantilever slab with the same effective span $l_0^c$ as the slab being analysed and can be calculated as $M_0^c = PL_0^c$) are presented along the Z-direction as shown in Figure 6. The effective span $l_0^c$ of a simply supported slab is regarded as the clear distance between the top flanges of two girders plus the slab depth according to Chinese, Japanese codes and AASHTO LRFD specification. While the effective span $l_0^c$ of a cantilever slab is taken as the distance between the load’s center line and the web’s center line according to AASHTO LRFD specification.

![Figure 6: Distribution of moment distribution along the longitudinal direction](image)

Figure 6 (a) shows that $M_s/M_0$ reaches its peak proportion value about 18% at section I which is much less than 70% and 80% recommended by codes. And $M_c/M_0$ is about 85% which is higher than 70% and 80%. Figure 6 (b) shows that $M_c^c/M_0^c$ is about 74%. It is indicated that the rotation constraints on slabs from steel girders are between the fixed constraints and the simply supported constraints, and sometimes the recommendations especially for continuous slabs from codes are too conservative for $M_s/M_0$ or a little dangerous for $M_c/M_0$. In addition, as a large part of cantilever moments can be transferred to the midspan, it is necessary for bridge designers to consider the negative moment effect at midspan caused by the live load at cantilever.

2.4 The impacts of headed studs spacing

In above analysis, it is assumed that there exists no slip between steel girders and concrete slabs. However, slabs are connected to steel girders by a group of connectors, bearing that the shear forces existing at the steel-concrete interface. To simulate actual conditions of the steel-concrete interfaces, the interaction between steel and concrete is simulated by “hard” contact in the normal direction and “Coulomb friction” model in the tangential direction with the friction coefficient $\mu = 0.4$ according to the previous research [13]. The stud connectors are simulated using the CARTESIAN connector element, which provides a connection between two nodes that allows independent behavior in three local Cartesian directions. The shear stiffness and tensile stiffness of the connector element are taken as 350 kN/mm and 250 kN/mm, respectively. To investigate the impacts of the headed studs spacing on the transverse moment distribution, three types of headed studs longitudinal and transverse spacing including 300mm×300mm, 200mm×200mm and 100mm×100mm are examined. The
results are shown in Figure 9.

![Figure 9: The impacts of studs spacing on transverse moment distribution](image)

Figure 9(a) shows that when slabs are loaded at midspan, with the enlargement of headed studs spacing, $M_c/M_0$ has an increasing tendency, while $M_s/M_0$ decreases. The “tie” model, which means the nodes in the steel-concrete interface are coupled to make no slip occurs between steel and concrete, has the maximum $M_s/M_0$ and the minimum $M_c/M_0$. Figure 9(b) shows when slabs are loaded at cantilever, $M_c/M_0^c$ has an increasing tendency with the enlargement of studs spacing and the “tie” model has the minimum $M_c/M_0^c$. It is indicated that the rotation constraints on slabs from steel girders become weaker with the enlargement of studs spacing and the strongest rotation constraints on slabs occur in the “tie” model. In addition, the maximum difference between models is less than 6%, it is indicated that the influence of the change of studs spacing and usage of “tie” constraint in finite element model is not obvious.

### 3 THEORETICAL MODEL STUDY

Section 1 in above finite element model can be simplified as a frame model supported by springs. As shown in Figure 10, a frame model with width $b$ is established and the bridge members are simplified as rod members rigidly connected to each other. The constraints from other sections around frame are simulated by horizontal and vertical springs to ensure the deformation coordination of the frame and the entire composite bridge[14]. The overall stiffness of the vertical springs can be obtained according to Eq. (1):

$$k_w = P_1 / \delta_w$$  \hspace{1cm} (1)

Where $P_1$ is the vertical load on the frame within width $b$; $\delta_w$ is the vertical deflection of the objective section under vertical load; the stiffness of two vertical springs $k_1, k_2$ can be taken as half of overall stiffness $k_w$.

Similarly, the overall stiffness of the transverse springs can be calculated according to Eq. (2):

$$k_u = P_2 / \delta_u$$  \hspace{1cm} (2)

Where $P_2$ is the transverse load on the frame within width $b$; $\delta_u$ is the transverse deflection of the objective section under transverse load; the stiffness of two transverse springs $k_3, k_4$ can be assigned from overall stiffness $k_u$ by the relative value of the transverse bending moment of inertia of the concrete slabs and cross-beam, respectively.
In order to reflect the relationship between the transverse moment distribution and the cross section characteristics of each member in the frame model, the linear flexural stiffness of the slabs, webs and cross-beams are defined as $c_i = E_i I_i / l$, $i_1 = E_i I_{i1} / h$ and $i_2 = E_i I_{i2} / l$ respectively. The ratio of slabs’ line flexural stiffness to webs’ is defined as $m_1 = i_1 / i_{i1}$, the ratio of slabs’ line flexural stiffness to cross-beams’ is defined as $m_2 = i_1 / i_{i2}$. Ignore the axial deformation while solving the statically indeterminate structure considering bending deformations are much larger than axial deformations of rod members in this frame model to obtain simplified expressions. Then the expressions are modified by coefficients to take the axial deformations into consideration. The $M_s$ at section I when slabs are loaded at midspan can be simply calculated as:

$$M_s = \alpha_1 P l_0 \left( 1 - \frac{6m_2 + 2m_1 + 4m_1 m_2 + m_1^2}{12m_2 + 4m_1 + 4m_1 m_2 + m_1^2} \right)$$

(3)

$\alpha_1$ is the coefficient to reflect the axial deformations of rod members, which is positively related to $k_3$ and $k_4$, indicating the stronger the transverse constraints on frame from nearby cross sections are, the larger the value of $M_s$ is. The value range of $\alpha_1$ is 0.96–0.99 from parameter analysis of $k_3$ and $k_4$ and can be taken as 1 conservatively. In Eq.(3), it is clear that $M_s$ is positively related to $m_1$ and $m_2$, which means the the maximum negative moment $M_s$ increases with the increase of the ratio of slabs’ line flexural stiffness to web’s and cross-beam’s.

Similarly, when slab is loaded at cantilever, $M_c$ could be simply calculated as:

$$M_c = \alpha_2 P l_0^c \left( \frac{m_1^2 + 4m_1 m_2}{m_1^2 + 4m_1 m_2 + 4m_1 + 12m_2} \right)$$

(4)

$\alpha_2$ is the coefficient to reflect the axial deformation of rod members, which is negative correlation of $k_3$ and $k_4$, indicating the stronger the transverse constraints on frame from nearby cross sections are, the smaller the $M_c$ is. The value range of $\alpha_2$ is 1.10–1.15 from parameter analysis of $k_3$ and $k_4$ and can be taken as 1.15 conservatively. In Eq. (4), the value of $M_c$ is negatively related to $m_1$ and $m_2$, which means the negative moment transferred to midspan from cantilever $M_c$ decreases with the increase of the ratio of slab’s line flexural stiffness to web’s and cross-beam’s.

By taking the width of frame $b_2 \leq 0.3m$, $I_{s1} = b l_{s1} / s_1$, $I_{s2} = b l_{s2} / s_2$ ($I_{s1}$ is the moment of inertia calculated according to the section showed in Figure13 and $s_1$ is the longitudinal spacing of vertical stiffener, $I_{s2}$ is the moment of inertia calculated according to the actual section of cross-beam and $s_2$ is the longitudinal spacing of cross-beam), frame model and
finite element models are applied to calculate $M_s/M_0$ and $M_c/M_0$ of models with different slabs and steel girders dimensions based on three actual bridges in Table 1. Figure 14 shows the results comparison calculated by Eqs. (3), (4) and finite element model. The maximum error between these two methods is less than 8%. The results of $M_s/M_0$ calculated by frame model are bigger than those calculated by FEA while the results of $M_c/M_0$ calculated by frame model are smaller than that calculated by method. So if the frame model is applied in bridge slabs design, the results can be conservative.

<table>
<thead>
<tr>
<th>Models</th>
<th>Slab’s thickness minimum/maximum (mm)</th>
<th>Slab’s effective span (mm)</th>
<th>Web’s height/thickness (mm)</th>
<th>Cross-beam’s height/width (mm)</th>
<th>Longitudinal spacing of vertical stiffener (mm)</th>
<th>Longitudinal spacing of cross-beam (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load at midspan</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>260/355</td>
<td>3750</td>
<td>1800/14</td>
<td>528/300</td>
<td>2000</td>
<td>4000</td>
</tr>
<tr>
<td>M3</td>
<td>240/400</td>
<td>4500</td>
<td>1300/20</td>
<td>400/300</td>
<td>3000</td>
<td>6000</td>
</tr>
<tr>
<td>M4</td>
<td>260/355</td>
<td>6000</td>
<td>2400/22</td>
<td>540/300</td>
<td>3500</td>
<td>7000</td>
</tr>
<tr>
<td>Load at cantilever</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>260/355</td>
<td>1850</td>
<td>1800/14</td>
<td>528/300</td>
<td>2000</td>
<td>4000</td>
</tr>
<tr>
<td>M5</td>
<td>240/400</td>
<td>2050</td>
<td>1300/20</td>
<td>400/300</td>
<td>3000</td>
<td>6000</td>
</tr>
<tr>
<td>M6</td>
<td>260/355</td>
<td>2250</td>
<td>2400/22</td>
<td>540/300</td>
<td>3500</td>
<td>7000</td>
</tr>
</tbody>
</table>

4 CONCLUSIONS
In this paper, the author conducted finite element analysis and theoretical analysis on the transverse moment distribution of deck slabs of twin girder cross-beam composite bridges. The following conclusions are obtained:

1. The rotation constraints on slabs from steel girders are between the fixed constraints and the simply supported constraints. When slabs are loaded at midspan, the value of the maximum negative moment is positively related to the transverse flexural stiffness of steel girders, and the peak value is at the section containing cross-beam. When slabs are loaded at cantilever, some negative moment will be transferred to the midspan slabs and the steel girders, and the value of the moment transferred to steel girders is positively related to the transverse flexural stiffness of steel girders.
2. With the enlargement of spacing of studs at the steel-concrete interface, the rotation constraints on slabs from steel girders become weaker. But the influence of the change of studs spacing and usage of “tie” constraint in finite element model is not obvious.

3. The frame model with springs proposed in this paper reveals the mechanism of how the structural characteristics of steel girders influence the transverse moment distribution of slabs. The results of moment distribution calculated by frame model are of acceptable accuracy compared with FEA.

REFERENCES
